

Chiral dynamics of the $\Lambda(1520)$ in coupled channels tested in the $K^- p \rightarrow \pi\pi\Lambda$ reaction

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Introduction

- Chiral Dynamics + Unitary Coupled Channels \Rightarrow Successful prediction of many $J^P = 1/2^-$ baryon resonances e.g. $N^*(1535)$, $\Lambda(1405)$, $\Lambda(1670)$, $\Sigma(1620)$, $\Xi(1620)$.
- *s-wave* scattering of the pion octet 0^- with the baryon decuplet $3/2^+$
 \Rightarrow Dynamical generation of $J^P = 3/2^-$ baryon resonances
- Some 4-star resonances which could be qualitatively described in this scheme are: $N^*(1520)$, $\Delta(1700)$, $\Lambda(1520)$, $\Sigma(1670)$, $\Sigma(1940)$, $\Xi(1820)$
 - *Kolomeitsev and Lutz, PLB 585(04)243*
 - *S.S., E. Oset, M.J. Vicente Vacas, NPA 750(2005)294*

Formulation

Lowest order chiral Lagrangian (*Decuplet - Octet interaction*)

$$\mathcal{L} = -i\bar{T}^\mu \not{p} T_\mu$$

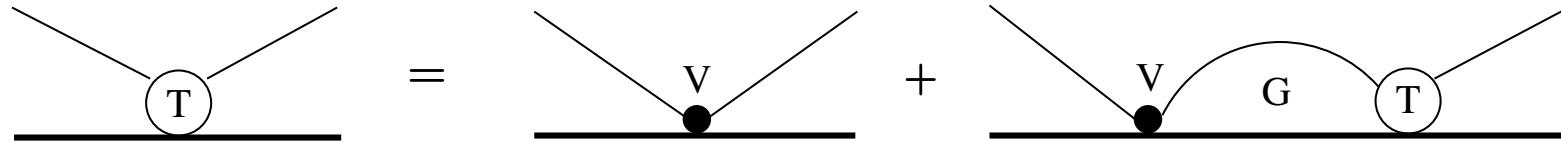
E. Jenkins et al, PLB 259 (91) 353

For a meson of incoming (outgoing) momenta $k(k')$ we get for the *s-wave* transition amplitudes,

$$V_{ij} = -\frac{1}{4f^2} C_{ij} (k^0 + k'^0)$$

This V is used as kernel of a coupled channels Bethe Salpeter equation

$$T = (1 - VG)^{-1} V$$



where G is the meson baryon loop function

Formulation

$$G_l = i2M_l \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}$$

Using dimensional regularization

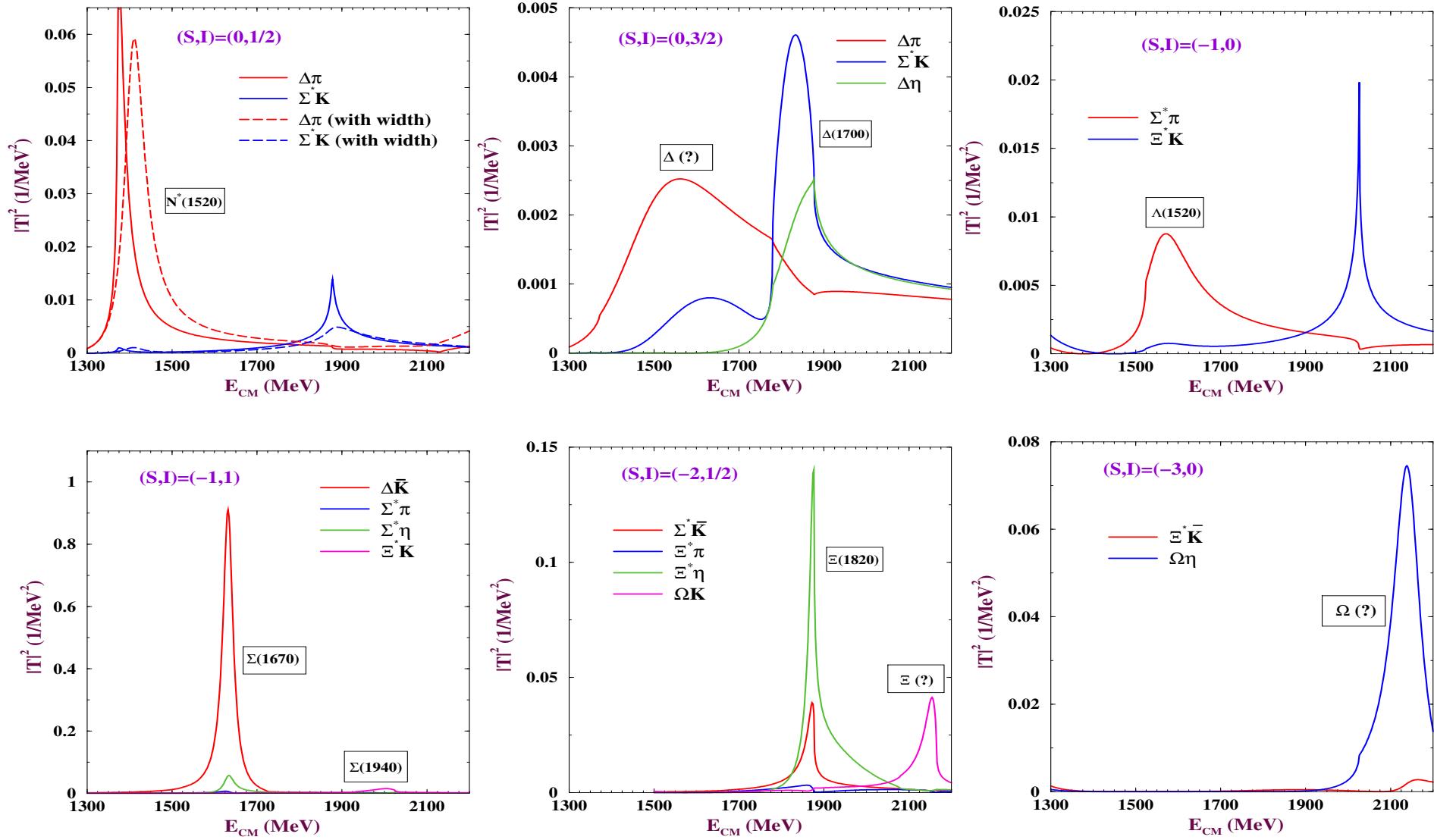
$$G_l = \frac{1}{16\pi^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \right. \\ \left. \frac{q_l}{\sqrt{s}} [\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s}) + \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \right. \\ \left. \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s})] \right\},$$

with unknown parameters $a_l \sim -2$ (corresponding to a cut-off of ~ 700 MeV).

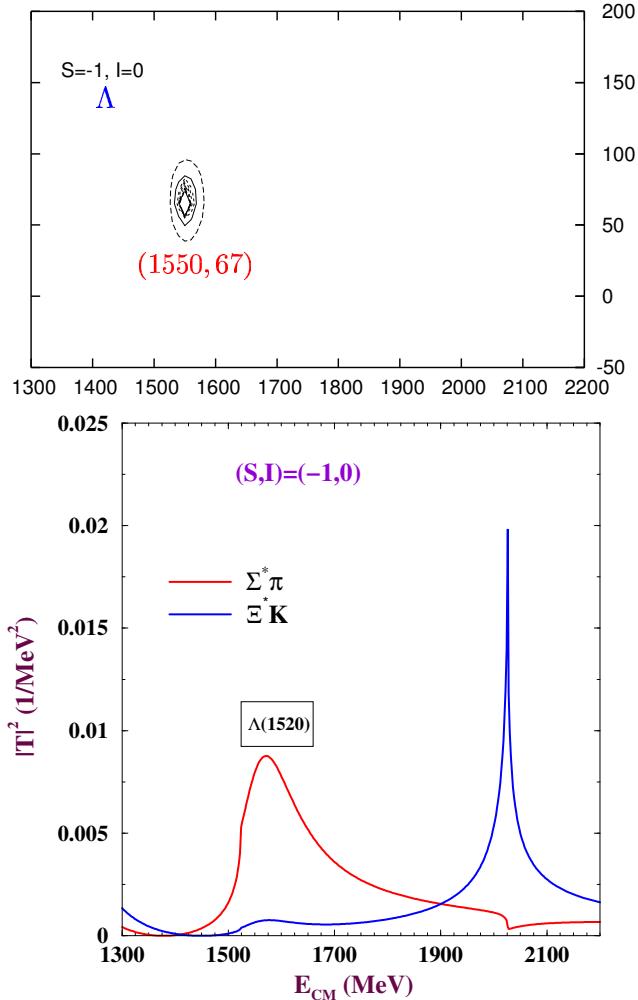
Solve the BS equation and look for poles of T in the 2nd Riemann sheet of the complex energy plane \Rightarrow resonances

Close to the pole (z_R) $T_{ij}(z) = \frac{g_i g_j}{z - z_R}$; residue \rightarrow couplings g_i \rightarrow partial decay widths

$J^P = \frac{3}{2}^-$ Resonances



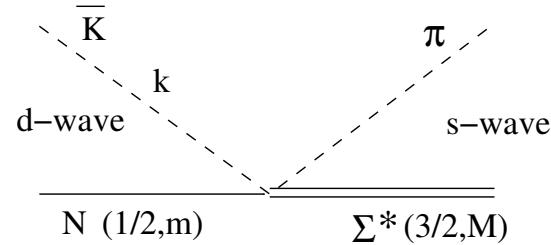
Results: $S = -1, I = 0$ (Λ)



- Pole at $1550 - i67$ MeV dynamically generated with $\pi\Sigma^*$ and $K\Xi^*$ in coupled channels
- Strong coupling to $\pi\Sigma^*$ channel – a quasibound state in this simple model
- About 30 MeV higher in mass and ~ 10 times broader than the nominal width (15.6 MeV)
- PDG gives large branching ratios to $\bar{K}N$ (45%) and $\pi\Sigma$ (42%) \implies must be included
- The lowest partial wave in which the channels $\bar{K}N$ and $\pi\Sigma$ can couple to spin parity $3/2^-$ is $L = 2$.
- Inclusion of the Σ^* width in G is important

Λ(1520)

Coupling of *d-wave* channels $\bar{K}N$ and $\pi\Sigma$:



$$-it_{\bar{K}N \rightarrow \pi\Sigma^*} = -i\gamma_{\bar{K}N} |\vec{k}|^2 \left[T^{(2)\dagger} \otimes Y_2(\hat{k}) \right]_{00}$$

where

$$\langle 3/2 \ M | T_\mu^{(2)\dagger} | 1/2 \ m \rangle = \mathcal{C}(1/2 \ 2 \ 3/2; m \ \mu \ M) \langle 3/2 | | T^{(2)\dagger} | | 1/2 \rangle$$

so that

$$-it_{\bar{K}N \rightarrow \pi\Sigma^*} = -i\gamma_{\bar{K}N} |\vec{k}|^2 \mathcal{C}(\frac{1}{2} \ 2 \ \frac{3}{2}; m, M-m) Y_{2,m-M}(\hat{k}) (-1)^{M-m} \sqrt{4\pi}$$

Λ(1520)

On-shell factorization of the vertices leads to the V matrix

	$\pi\Sigma^*$	$K\Xi^*$	$\bar{K}N$	$\pi\Sigma$
$\pi\Sigma^*$	$C_{11}(k_1^0 + k_1^0)$	$C_{12}(k_1^0 + k_2^0)$	$\gamma_{13} q_3^2$	$\gamma_{14} q_4^2$
$K\Xi^*$	$C_{21}(k_2^0 + k_1^0)$	$C_{22}(k_2^0 + k_2^0)$	0	0
$\bar{K}N$	$\gamma_{13} q_3^2$	0	$\gamma_{33} q_3^4$	$\gamma_{34} q_3^2 q_4^2$
$\pi\Sigma$	$\gamma_{14} q_4^2$	0	$\gamma_{34} q_3^2 q_4^2$	$\gamma_{44} q_4^4$

where $C_{11} = -\frac{1}{4f^2}$, $C_{22} = -\frac{3}{4f^2}$ and $C_{12} = C_{21} = -\frac{\sqrt{6}}{4f^2}$

$$q_i = \frac{1}{2\sqrt{s}} \sqrt{[s - (M_i + m_i)^2][s - (M_i - m_i)^2]} \quad k_i^0 = \frac{s - M_i^2 + m_i^2}{2\sqrt{s}}$$

we can continue with the formalism as in ordinary s -wave scattering.

$\Lambda(1520)$

To take the $\pi\Sigma^*$ width into account we fold G with the **spectral function** of the Σ^* :

$$G_{\pi\Sigma^*}(\sqrt{s}, M_{\Sigma^*}, m_\pi) \rightarrow \int_{M_{\Sigma^*} - 2\Gamma_0}^{M_{\Sigma^*} + 2\Gamma_0} d\sqrt{s'} \frac{-1}{\pi} \text{Im} \left[\frac{1}{\sqrt{s'} - M_{\Sigma^*} + i\Gamma_{\Sigma^*}(s')/2} \right] \times G_{\pi\Sigma^*}(\sqrt{s}, \sqrt{s'}, m_\pi)$$

where

$$\begin{aligned} \Gamma_{\Sigma^*}(s') = \Gamma_0 & \left(0.88 \frac{q^3(s', M_\Lambda^2, m_\pi^2)}{q^3(M_{\Sigma^*}^2, M_\Lambda^2, m_\pi^2)} \Theta(\sqrt{s'} - M_\Lambda - m_\pi) \right. \\ & \left. + 0.12 \frac{q^3(s', M_\Sigma^2, m_\pi^2)}{q^3(M_{\Sigma^*}^2, M_\Sigma^2, m_\pi^2)} \Theta(\sqrt{s'} - M_\Sigma - m_\pi) \right), \end{aligned}$$

Λ(1520)

Using V as the kernel and the loop function G we use the coupled channel BS equation to get the amplitudes T_{ij} for $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \pi\Sigma$

From a fit to the experimental amplitudes \tilde{T}_{ij} we find the unknown parameters $\gamma_{13}, \gamma_{14}, \gamma_{33}, \gamma_{34}, \gamma_{44}$ in the V matrix and the subtraction constants a_0, a_2 in G

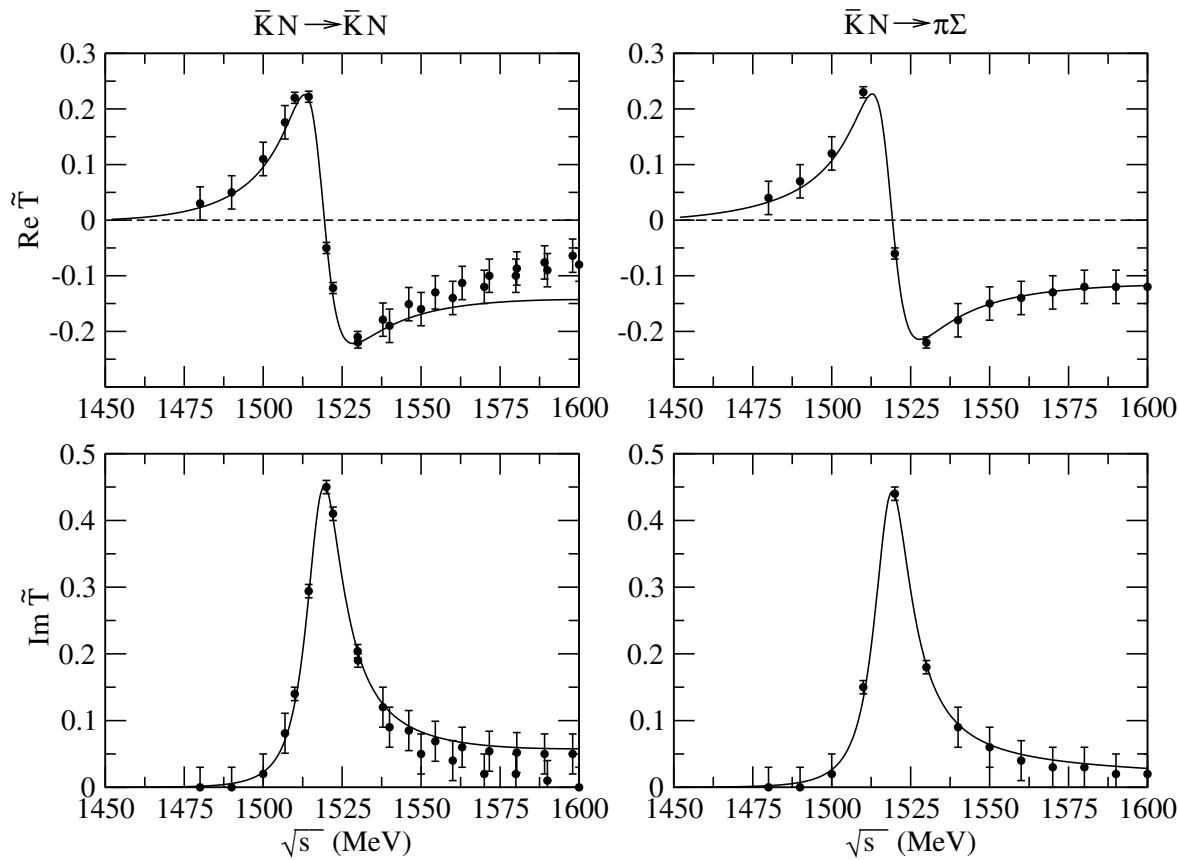
$$\tilde{T}_{ij}(\sqrt{s}) = -\sqrt{\frac{M_i q_i}{4\pi\sqrt{s}}} \sqrt{\frac{M_j q_j}{4\pi\sqrt{s}}} T_{ij}(\sqrt{s})$$

$$B_i = \frac{\Gamma_i}{\Gamma} = \text{Im} \tilde{T}_{ii}(\sqrt{s} = M_{\Lambda(1520)})$$

We get the following values

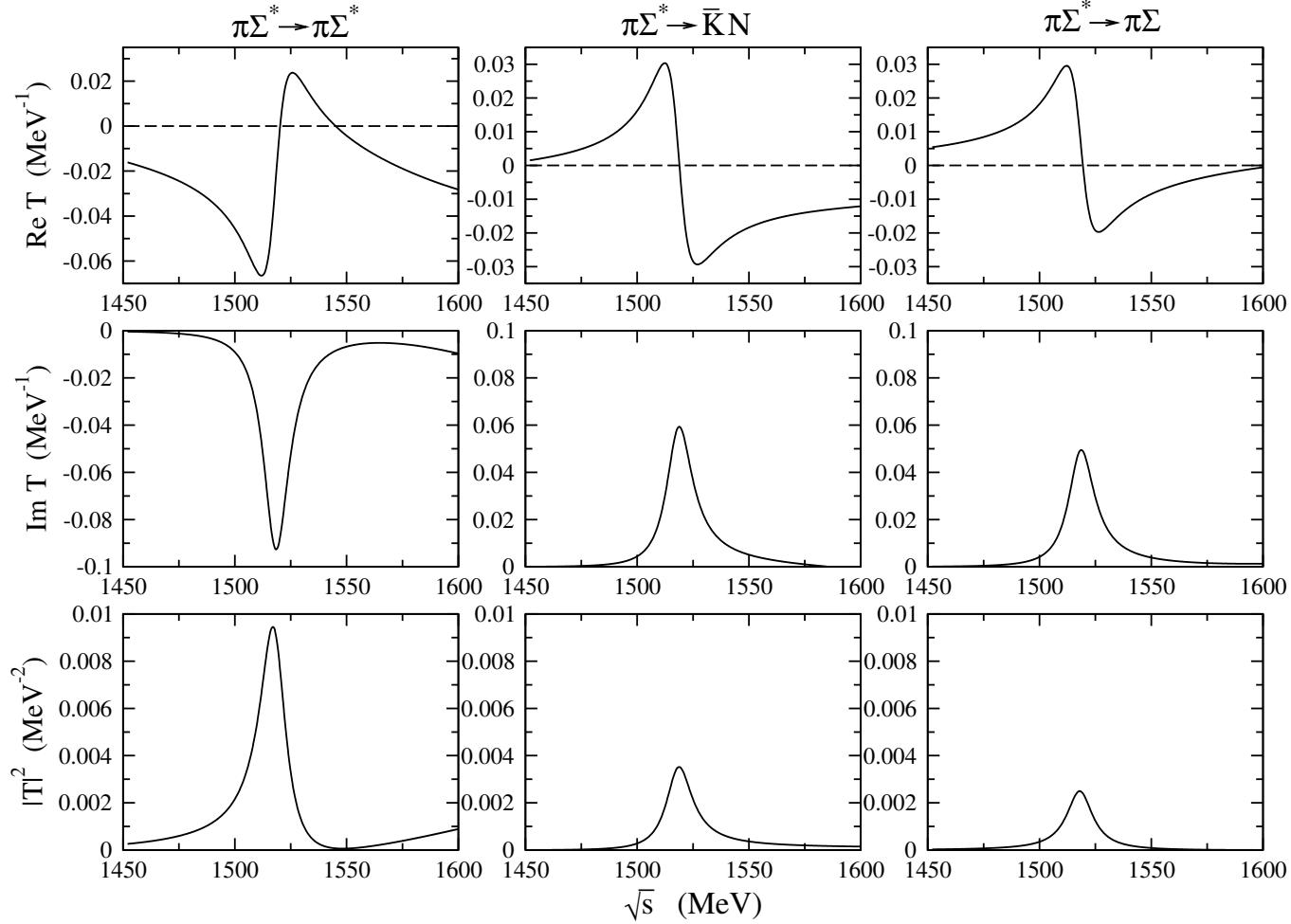
a_0	a_2	$\gamma_{13} (\text{MeV}^{-3})$	$\gamma_{14} (\text{MeV}^{-3})$	$\gamma_{33} (\text{MeV}^{-5})$	$\gamma_{44} (\text{MeV}^{-5})$	$\gamma_{34} (\text{MeV}^{-5})$
-1.8	-8.1	0.98×10^{-7}	1.1×10^{-7}	-1.7×10^{-12}	-0.7×10^{-12}	-1.1×10^{-12}

$\Lambda(1520)$



Fit to the experimental amplitudes. Left column: $\bar{K}N \rightarrow \bar{K}N$; right column: $\bar{K}N \rightarrow \pi\Sigma$.
 Experimental data from G. P. Gopal *et al.* NPB119 (1977) 362 and M. Alston-Garnjost *et al.* PRD18 (1978) 182

$\Lambda(1520)$



From left to right: Unitary amplitudes for $\pi\Sigma^* \rightarrow \pi\Sigma^*$, $\pi\Sigma^* \rightarrow \bar{K}N$ and $\pi\Sigma^* \rightarrow \pi\Sigma$.

$\Lambda(1520)$

Close to the peak the amplitudes are given by

$$T_{ij}(\sqrt{s}) = \frac{g_i g_j}{\sqrt{s} - M_{\Lambda(1520)} + i\Gamma_{\Lambda(1520)}/2}$$

from where we calculate the couplings to the different channels:

$$g_i g_j = -\frac{\Gamma_{\Lambda(1520)}}{2} \frac{|T_{ij}(M_{\Lambda(1520)})|^2}{Im[T_{ij}(M_{\Lambda(1520)})]},$$

We get

g_1	g_2	g_3	g_4
0.91	-0.29	-0.54	-0.45

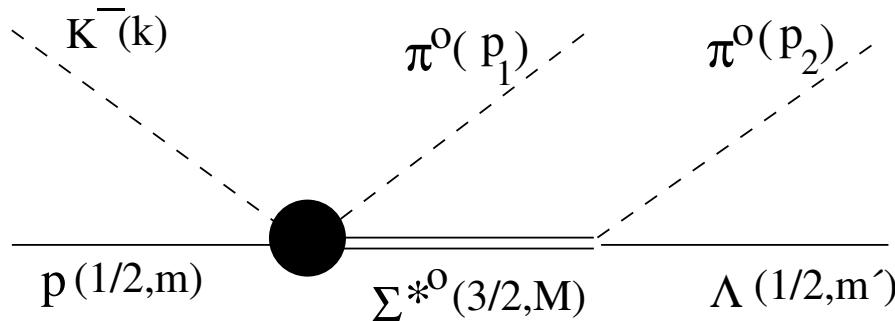
The partial decay widths can be obtained from

$$\Gamma_i = \frac{g_i^2}{2\pi} \frac{M_i}{M_{\Lambda(1520)}} q_i$$

$\Lambda(1520)$

Predictions:

The reaction $K^- p \rightarrow \pi^0 \Sigma^{*0}(1385) \rightarrow \pi^0 \pi^0 \Lambda(1116)$

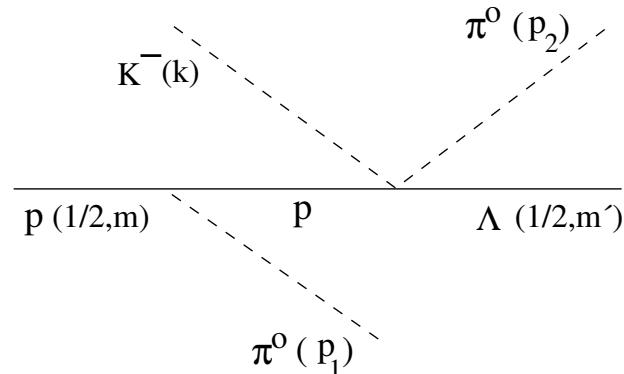


$$-it(\vec{p}_1, \vec{p}_2) = \frac{-iT_{\bar{K}N \rightarrow \pi\Sigma^*}}{3\sqrt{2}} \frac{f_{\Sigma^*\pi\Lambda}/m_\pi}{M_R - M_{\Sigma^*} + i\Gamma_{\Sigma^*}(M_R)/2} \begin{Bmatrix} -2p'_{2z} & m' = +1/2 \\ p'_{2x} + ip'_{2y} & m' = -1/2 \end{Bmatrix}$$

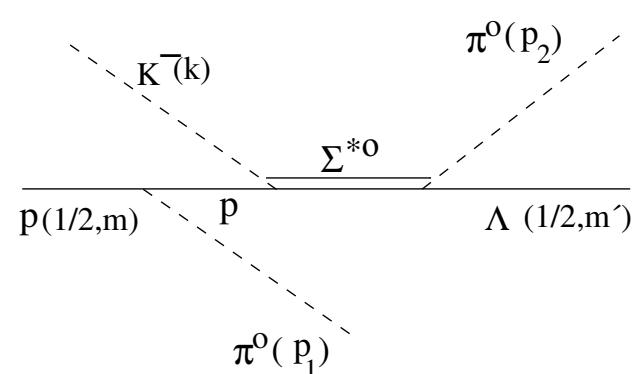
Symmetrize the amplitude \rightarrow three-body phase space \rightarrow cross-section

$\Lambda(1520)$

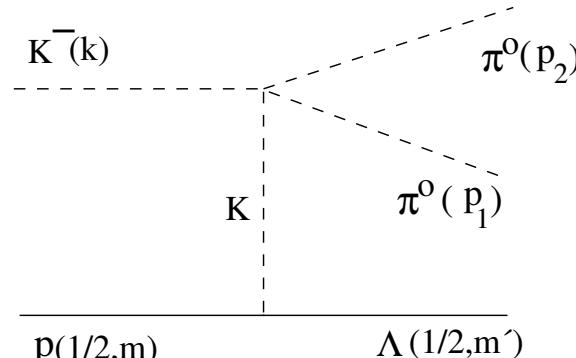
We also add the following conventional diagrams



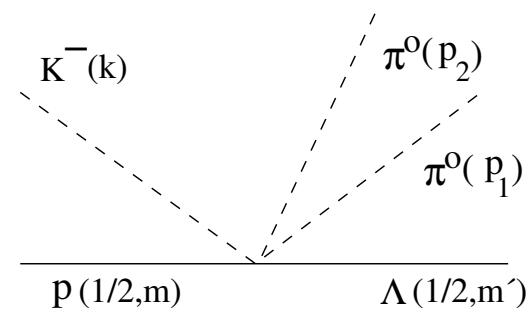
(a)



(b)

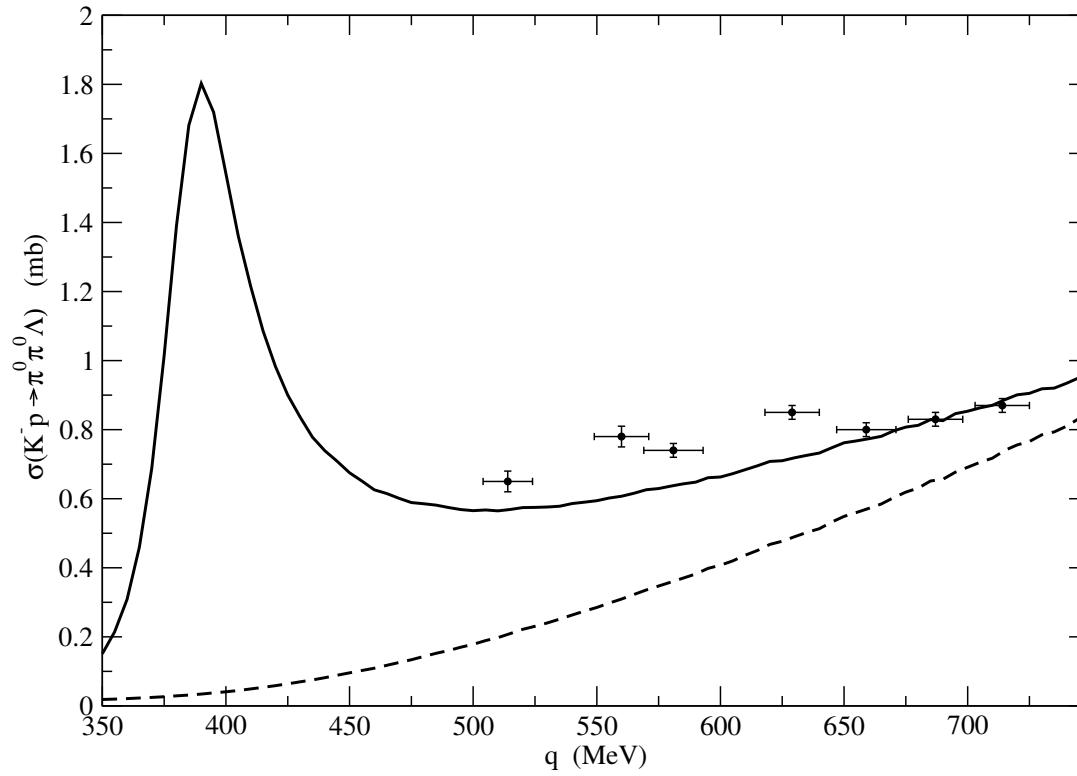


(a)



(b)

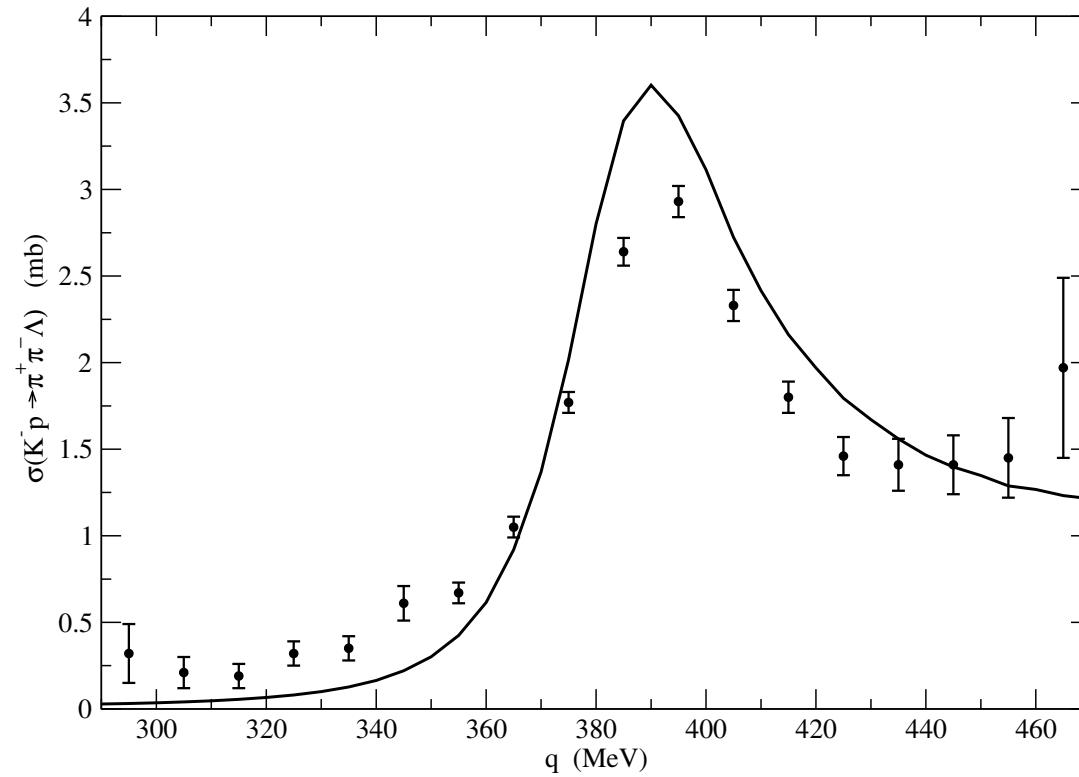
$\Lambda(1520)$



$K^- p \rightarrow \pi^0 \pi^0 \Lambda$ cross section (mb) vs p_{lab} (MeV) of K^-

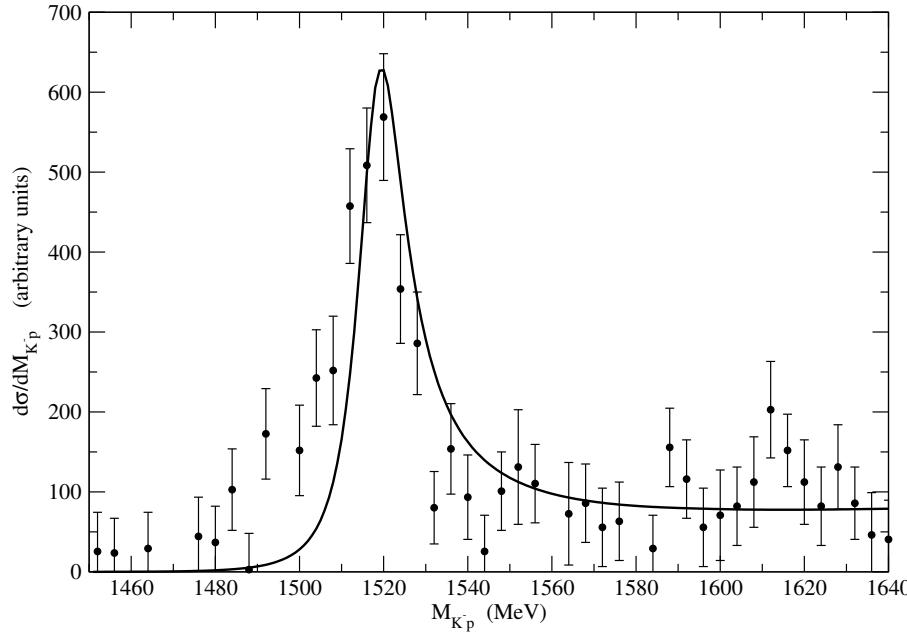
Experimental data from S. Prakhov *et al.*, PRC69 (2004) 042202

$\Lambda(1520)$



$K^- p \rightarrow \pi^+ \pi^- \Lambda$ cross section (mb) vs p_{lab} (MeV) of K^-
Experimental data from T. S. Mast *et al.*, PRD7 (1973) 5

$\Lambda(1520)$



$K^- p$ invariant mass distribution for the $\gamma p \rightarrow K^+ K^- p$ reaction with photons in the range $E_\gamma = 2.8 - 4.8$ GeV.

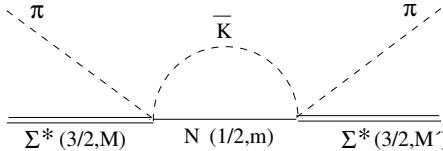
Experimental data from D. P. Barber *et al.*, Z. Phys. C7 (1980) 17

Summary

- The $\Lambda(1520)$ is generated dynamically in our unitary coupled channel approach with the $\pi\Sigma^*$ and $K\Xi^*$ channels in *s-wave* and the $\bar{K}N$ and $\pi\Sigma$ channels in *d-wave*.
- The $\Lambda(1520)$ couples strongly to the $\pi\Sigma^*$ channel though the branching ratios to the $\bar{K}N$ and $\pi\Sigma$ channels are much larger
- The prediction of the $\bar{K}N \rightarrow \pi\Sigma^*$ amplitude and consequently the absolute cross section in the reactions $K^- p \rightarrow \Lambda\pi^0\pi^0$ and $K^- p \rightarrow \Lambda\pi^+\pi^-$ agree fairly well with experimental data.

Extras

Λ(1520)



$$\begin{aligned}
 T_2 &= i \int \frac{d^4 q}{(2\pi)^4} G_N D_{\bar{K}} 4\pi \\
 &\gamma_{\bar{K}N} |\vec{q}|^2 \sum_m \mathcal{C}(1/2 \ 2 \ 3/2; m, M' - m) Y_{2,m-M'}(\hat{q}) (-1)^{M' - m} \\
 &\gamma_{\bar{K}N} |\vec{q}|^2 \mathcal{C}(1/2 \ 2 \ 3/2; m, M - m) Y_{2,m-M}^*(\hat{q}) (-1)^{M - m}
 \end{aligned}$$

- Angular integration of the two $Y \rightarrow \delta_{MM'}$
- Orthogonality of the Clebsch Gordan coefficients
- On-shell factorization of vertex

$$T_2 = [\gamma_{\bar{K}N} q_{on}]^2 \times G_{\bar{K}N} = V_{\pi\Sigma^* \rightarrow \bar{K}N} G_{\bar{K}N} V_{\bar{K}N \rightarrow \pi\Sigma^*}$$

Formulation

Example: $S = -1, Q = 0$

	$\Delta^0 \bar{K}^0$	$\Delta^+ K^-$	$\Sigma^* - \pi^+$	$\Sigma^{*0} \pi^0$	$\Sigma^{*0} \eta$	$\Sigma^{*+} \pi^-$	$\Xi^* - K^+$	$\Xi^{*0} K^0$
$\Delta^0 \bar{K}^0$	2	2	-1	1	$-\sqrt{3}$	0	0	0
$\Delta^+ K^-$		2	0	-1	$-\sqrt{3}$	-1	0	0
$\Sigma^* - \pi^+$			2	2	0	0	2	0
$\Sigma^{*0} \pi^0$				0	0	-2	1	-1
$\Sigma^{*0} \eta$					0	0	$\sqrt{3}$	$\sqrt{3}$
$\Sigma^{*+} \pi^-$						2	0	2
$\Xi^* - K^+$							2	-1
$\Xi^{*0} K^0$								2

$$|\Sigma^* \pi I=0\rangle = \sqrt{\frac{1}{3}} |\Sigma^{*+} \pi^-\rangle - \sqrt{\frac{1}{3}} |\Sigma^{*0} \pi^0\rangle - \sqrt{\frac{1}{3}} |\Sigma^* - \pi^+\rangle;$$

$$|\Xi^* K I=0\rangle = \sqrt{\frac{1}{2}} |\Xi^{*0} K^0\rangle - \sqrt{\frac{1}{2}} |\Xi^* - K^+\rangle$$

to finally get for $I=0$

	$\Sigma^* \pi$	$\Xi^* K$
$\Sigma^* \pi$	4	$\sqrt{6}$
$\Xi^* K$		3

N/D Method

Unitarity states that, above threshold,

$$[Imt^{-1}(s)]_{ij} = -\frac{q_i}{4\pi} \frac{M_i}{\sqrt{s}} \delta_{ij} = ImG(s)$$

Using a subtracted dispersion relation

$$t^{-1}(s) = -G(s) + V^{-1}(s)$$

where $G(S)$ contains an arbitrary subtraction constant and V^{-1} accounts for contact terms which remain at tree level when $G = 0$.

The above equation can be cast as

$$t = [1 - VG]^{-1} = V + VGt$$

Formulation

We will consider only the **s-wave** part of the interaction and the **non-relativistic** limit, so that

$$\bar{u}(p', s') \gamma^\nu u(p, s) = \delta^{\nu 0} \delta_{ss'} + \mathcal{O}(|\vec{p}|/M)$$

$$\sum_{\lambda', s'} \sum_{\lambda, s} \mathcal{C}(1 \frac{1}{2} \frac{3}{2}; \lambda' s' s_\Delta) e_\mu^*(p', \lambda') \mathcal{C}(1 \frac{1}{2} \frac{3}{2}; \lambda s s_\Delta) e^\mu(p, \lambda) \delta_{ss'} = -1 + \mathcal{O}(|\vec{p}|^2/M^2).$$

$$\mathcal{L} = 3iTr\{\bar{T} \cdot T \Gamma^{0T}\}$$

$$(\bar{T} \cdot T)_d^a = \sum_{b,c} \bar{T}^{abc} T_{dbc}; \quad \Gamma^\nu = \frac{1}{4f^2} (\Phi \partial^\nu \Phi - \partial^\nu \Phi \Phi).$$

$$\begin{aligned} T^{111} &= \Delta^{++}, T^{112} = \frac{1}{\sqrt{3}} \Delta^+, T^{122} = \frac{1}{\sqrt{3}} \Delta^0, T^{222} = \Delta^-, T^{113} = \frac{1}{\sqrt{3}} \Sigma^{*+}, \\ T^{123} &= \frac{1}{\sqrt{6}} \Sigma^{*0}, T^{223} = \frac{1}{\sqrt{3}} \Sigma^{*-}, T^{133} = \frac{1}{\sqrt{3}} \Xi^{*0}, T^{233} = \frac{1}{\sqrt{3}} \Xi^{*-}, T^{333} = \Omega^-. \end{aligned}$$

Formulation

$SU(3)$ decomposition: $8 \otimes 10 = 8 \oplus 10 \oplus 27 \oplus 35$

Projection on to $SU(3)$ basis: $C_{\alpha\beta} = \sum_{i,j} \langle i, \alpha \rangle C_{ij} \langle j, \beta \rangle$

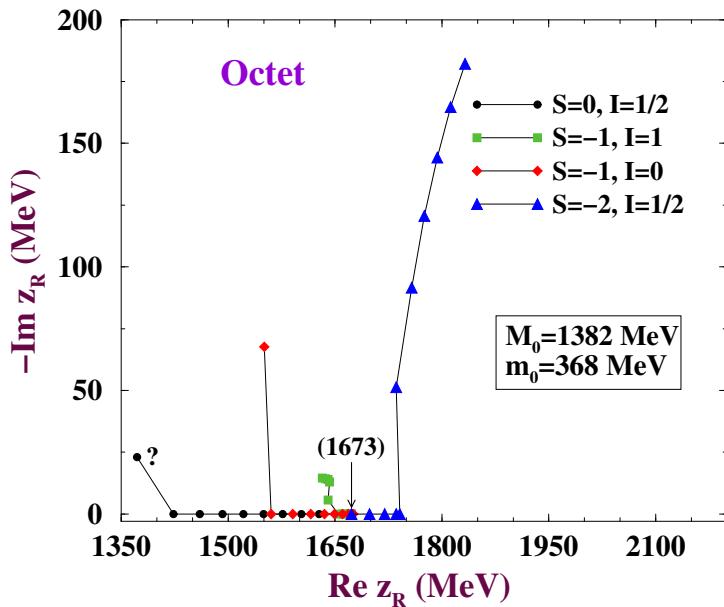
Strength is proportional to: $C_{\alpha\beta} = \text{diag}(6, 3, 1, -3)$

- strong attraction in octet, followed by decuplet
- weak attraction in 27, repulsion in 35

We then solve the BS equation and look for poles in the complex plane. In the $SU(3)$ limit we get two poles, one each for the octet and decuplet representations

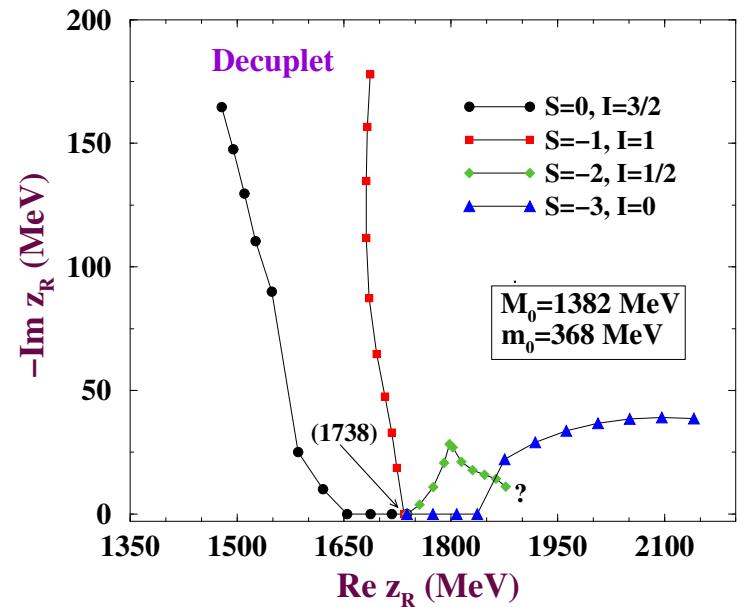
Results: Trajectories of Poles in the Complex Plane

Two bound states in the $SU(3)$ limit



Break $SU(3)$ symmetry gradually

$$\begin{aligned} M_i(x) &= M_0 + x(M_i - M_0) \\ m_i^2(x) &= m_0^2 + x(m_i^2 - m_0^2) \\ 0 \leq x \leq 1 \end{aligned}$$

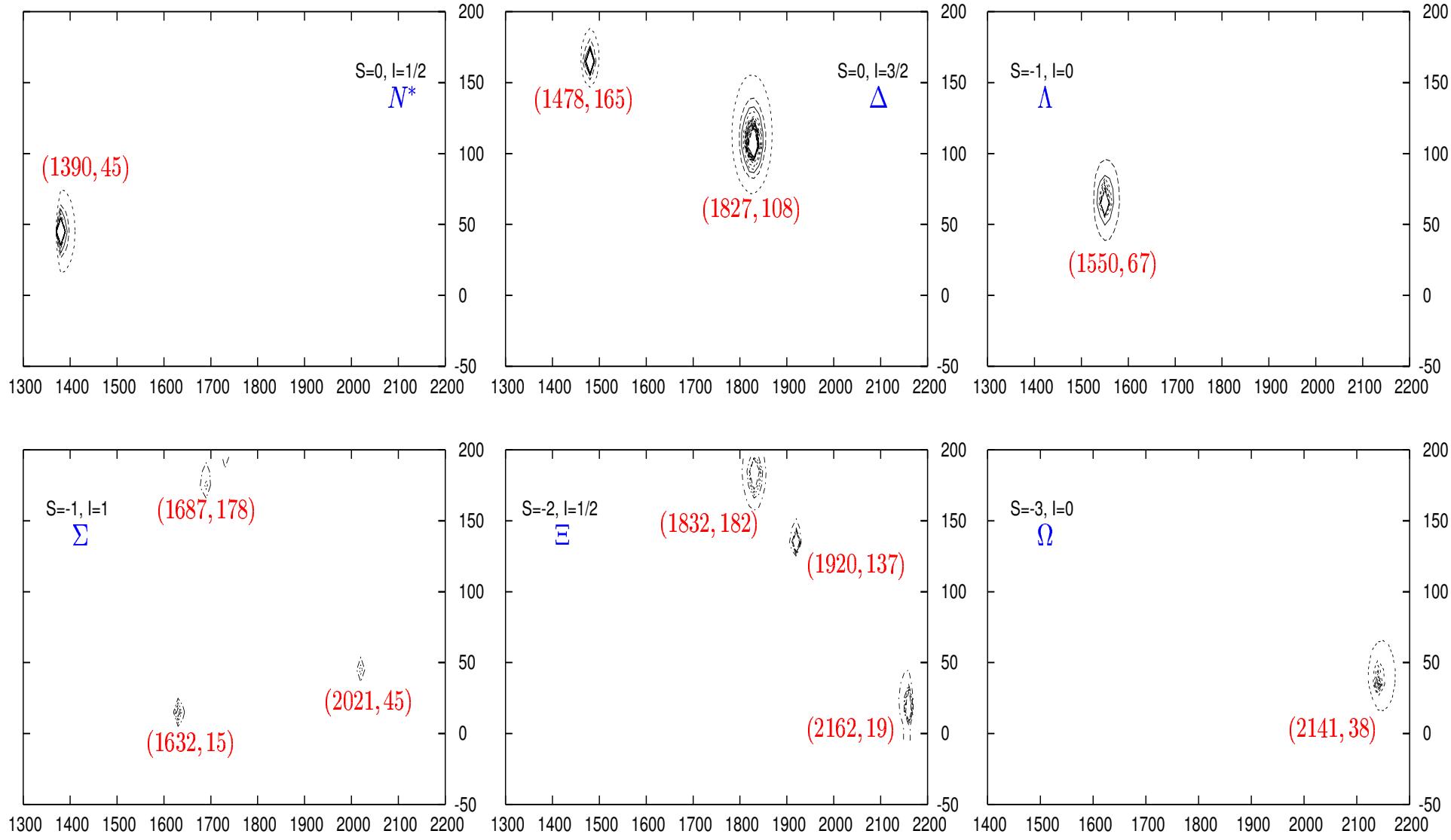


Close to the pole (z_R)

$$T_{ij}(z) = \frac{g_i g_j}{z - z_R}$$

residue \rightarrow couplings g_i

Poles in the Complex Plane



Couplings of Λ various to channels

z_R	1550 – $i67$	
	g_i	$ g_i $
$\Sigma^* \pi$	2.0 – $i1.5$	2.5
$\Xi^* K$	0.9 – $i0.8$	1.2

$\Lambda(1520)$

The actual amplitudes are given by

$$t_{\pi\Sigma^*\rightarrow\pi\Sigma^*} = T_{\pi\Sigma^*\rightarrow\pi\Sigma^*}$$

$$t_{K\Xi^*\rightarrow K\Xi^*} = T_{K\Xi^*\rightarrow K\Xi^*}$$

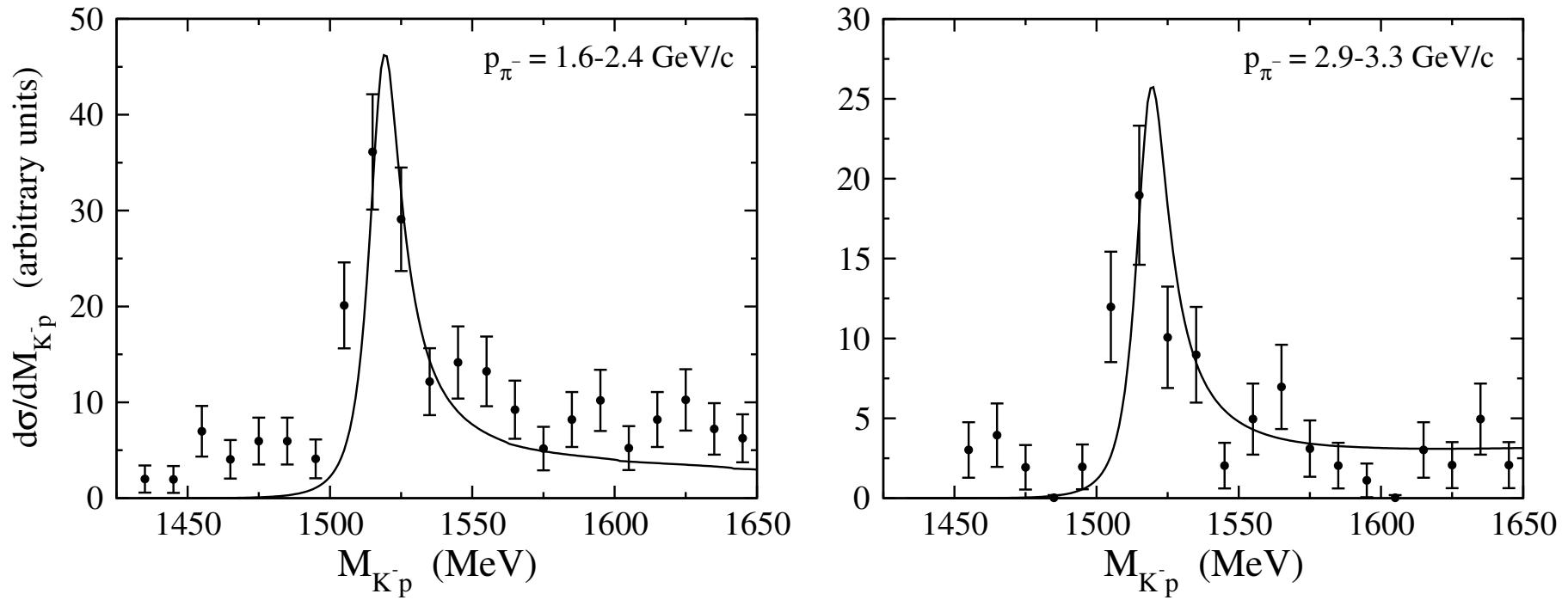
$$t_{\bar{K}N\rightarrow\pi\Sigma^*} = T_{\bar{K}N\rightarrow\pi\Sigma^*} \mathcal{C}\left(\frac{1}{2} 2 \frac{3}{2}; m, M-m\right) Y_{2,m-M}(\hat{k}) (-1)^{M-m} \sqrt{4\pi}$$

$$t_{\pi\Sigma\rightarrow\pi\Sigma^*} = T_{\pi\Sigma\rightarrow\pi\Sigma^*} \mathcal{C}\left(\frac{1}{2} 2 \frac{3}{2}; m, M-m\right) Y_{2,m-M}(\hat{k}) (-1)^{M-m} \sqrt{4\pi}$$

$$t_{\bar{K}N\rightarrow\bar{K}N} = T_{\bar{K}N\rightarrow\bar{K}N} \sum_M \mathcal{C}\left(\frac{1}{2} 2 \frac{3}{2}; m, M-m\right) Y_{2,m-M}(\hat{k}) \cdot$$

$$\cdot \mathcal{C}\left(\frac{1}{2} 2 \frac{3}{2}; m', M-m'\right) Y_{2,m'-M}^*(\hat{k}') (-1)^{m'-m} 4\pi .$$

Λ(1520)



K^-p invariant mass distribution for the $\pi^-p \rightarrow K^0 K^-p$ reaction. Left: pions in the range $p_{\pi^-} = 1.6 - 2.4 \text{ GeV}/c$; right: $p_{\pi^-} = 2.9 - 3.3 \text{ GeV}/c$.

Experimental data from O. I. Dahl *et al.*, Phys. Rev. **163** (1967) 1377